Stable Solutions Of Elliptic Partial Differential Equations Monographs And Surveys In Pure And Applied Mathematics

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Encyclopaedia of Mathematics Michiel Hazewinkel 2013-12-01 This ENCYCLOPAEDIA OF MATHEMATICS aims to be a reference work for all parts of mathe matics. It is a translation with updates and editorial comments of the Soviet Mathematical Encyclopaedia published by 'Soviet Encyclopaedia Publishing House' in five volumes in 1977-1985. The annotated translation consists of ten volumes including a special index volume. There are three kinds of articles in this ENCYCLOPAEDIA. First of all there are survey-type articles dealing with the various main directions in mathematics (where a rather fine subdivi sion has been used). The main requirement for these articles has been that they should give a reasonably complete up-to-date account of the current state of affairs in these areas and that they should be maximally accessible. On the whole, these articles should be understandable to mathematics students in their first specialization years, to graduates from other mathematical areas and, depending on the specific subject, to specialists in other domains of science, en gineers and teachers of mathematics. These articles treat their material at a fairly general level and aim to give an idea of the kind of problems, techniques and concepts involved in the area in question. They also contain background and motivation rather than precise statements of precise theorems with detailed definitions and technical details on how to carry out proofs and constructions. The second kind of article, of medium length, contains more detailed concrete problems, results and techniques.
Numerical Methods for Nonlinear Elliptic Partial Differential Equations Tiago Miguel Saldanha Salvador 2017 "The goal of this thesis is to widen the class of provably convergent schemes for elliptic partial differential equations (PDEs) and improve their accuracy. We accomplish this by building on the theory of Barles and Souganidis, and its extension by Froese and Oberman to build monotone and filtered schemes.The first problem considered is the widely studied class of first order Hamilton-Jacobi (HJ) equations. The goal is to construct provably convergent accurate schemes, together with an efficient solver, by making use of the large number of discretizations and solvers already available. To this end, we build filtered schemes, whose main idea is to blend a stable monotone convergent scheme with an accurate scheme while retaining the advantages of both: stability and convergence of the former, and higher accuracy of the latter. Indeed, we are able to build schemes which are second, third, and fourth order accurate in one dimension, as well as schemes that are second order accurate in two dimensions. Moreover, the schemes are explicit, allowing us to use the easy-to-implement fast sweeping method. Using different accurate schemes (e.g. from standard centred differences, higher order upwinding and ENO interpolation), the accuracy of the filtered schemes is validated with computational results for the eikonal equation, as well as other HJ equations (both in one and two dimensions).The second problem considered is the 2-Hessian equation, a fully nonlinear PDE related to the intrinsic curvature for three-dimensional manifolds. The goal is to build numerical methods to compute its solution on a bounded domain given prescribed boundary data. We propose two distinct methods. The first is provably convergent to the unique viscosity solution. The second has higher accuracy and converges in practice, but lacks a formal proof of convergence. The PDE is elliptic on a restricted set of functions: a convexity-type constraint is needed for the ellipticity of the PDE operator, which poses additional difficulties when building the numerical methods. Solutions with both discretizations are obtained using Newton's method. Computational results are presented on a number of exact solutions which range in regularity from smooth to non-differentiable, and in shape from convex to non-convex.The third and last problem is to build a provably convergent scheme for the nonlinear PDE that governs the motion of level sets by affine curvature. It is closely related to mean curvature but exhibits instabilities not found in the former. These instabilities and the lack of regularity of the affine curvature operator posed unexpected and additional difficulties in building monotone schemes. A standard finite difference scheme is proposed and an example that illustrates its nonlinear instability is given. We build provably convergent monotone finite difference schemes. Numerical experiments demonstrate the accuracy and stability of the discretization, as well as the fact that our approximate solutions capture the affine invariance and morphological properties of the evolution." --
On Some Applied Problems Using Nonlinear Elliptic Partial Differential Equations Christopher Finlay 2020 "This dissertation studies several applied mathematical problems which broadly fall under the modelling framework of nonlinear elliptic partial differential equations (PDEs). Solutions to these problems are placed in the analytic setting of the theory of viscosity solutions; convergence of corresponding numerical solutions rely on the monotone schemes using the proof technique of Barles and Souganidis.The first two chapters study the problem of homogenization of nonlinear elliptic PDEs: find a macroscopic operator and corresponding solutionthat captures the behaviour of a rapidly varying microscopic operator, on broad scales. This is done through duality theory; we approximately solve an equivalent dual problem, which provides bound(s) on the true homogenized operator. Numerical experiments show that these approximate homogenized operators are quite accurate; in some cases error bounds are available using semi-concavity estimates.The third chapter develops monotone finite difference schemes for nonlinear elliptic PDEs on point clouds. To date, most numerical methods for theseequations have been on regular grids; motivated by the work of Froese this chapter extends monotone finite difference schemes to point clouds. The schemes rely on linear interpolation of neighbouring points in barycentric coordinates. Our schemes are of higher accuracy than previously available on both regular grids and point clouds. We prove consistency and stability of the schemes and provide several numerical examples in 2D.The final chapter explores the problem of adversarial examples in image classification. These are small perturbations of an input image which cause aclassifier to fail where a human would easily succeed. We address this problem using gradient regularization, which is inextricably linked to Tikhonov regularization in inverse problems and the p-Laplacian. We provide bounds on the minimum distance necessary to perturb an image adversarially, and show that gradient regularization improves these bounds. Moreover we implement gradient regularization in a scaleable fashion, using finite differences. This allows for quick training of adversarially robust models on very large datasets, which was previously intractable using prior methods"--
Partial Differential Equations of Parabolic Type Avner Friedman 2013-08-16 With this book, even readers unfamiliar with the field can acquire sufficient background to understand research literature related to the theory of parabolic and elliptic equations. 1964 edition.
2018 MATRIX Annals Jan de Gier 2020-04-07 MATRIX is Australia's international and residential mathematical research institute. It facilitates new collaborations and mathematical advances through intensive residential research programs, each 1-4 weeks in duration. This book is a scientific record of the eight programs held at MATRIX in 2018: - Non-Equilibrium Systems and Special Functions - Algebraic Geometry, Approximation and Optimisation - On the Frontiers of High Dimensional Computation - Month of Mathematical Biology - Dynamics, Foliations, and Geometry In Dimension 3 - Recent Trends on Nonlinear PDEs of Elliptic and Parabolic Type - Functional Data Analysis and Beyond - Geometric and Categorical Representation Theory The articles are grouped into peer-reviewed contributions and other contributions. The peer-reviewed articles present original results or reviews on a topic related to the MATRIX program; the remaining contributions are predominantly lecture notes or short articles based on talks or activities at MATRIX.
Stability of Solutions of Integrable Partial Differential Equations Jeremy Upsal 2020 Stability analysis for solutions of partial differential equations (PDEs) is important for determining the applicability of a model to the physical world. Establishing stability for PDE solutions is often significantly more challenging than for ordinary differential equation solutions. This task becomes tractable for PDEs possessing a Lax pair. In this dissertation, I provide a general framework for computing large parts of the Lax spectrum for periodic and quasiperiodic solutions of a general class of PDEs possessing a Lax pair. This class consists of the AKNS hierarchy admitting a common reduction and generalizations. I then relate the Lax spectrum to the stability spectrum using the squared-eigenfunction connection. Using this, I demonstrate that the subset of the real line which is part of the Lax spectrum maps to stable elements of the linearization. Several examples that demonstrate the direct applicability of this work are provided. One example is worked out in detail: the stability analysis for the elliptic solutions of the focusing nonlinear Schrödinger (NLS) equation. For the NLS equation, I go further by establishing orbital stability of the elliptic solutions with respect to a class of perturbations of integer multiples of the period of the solution.
An Introduction To Viscosity Solutions for Fully Nonlinear PDE with Applications to Calculus of Variations in L_∞ Nikos Katzourakis 2014-11-26 The purpose of this book is to give a quick and elementary, yet rigorous, presentation of the rudiments of the so-called theory of Viscosity Solutions which applies to fully nonlinear 1st and 2nd order Partial Differential Equations (PDE). For such equations, particularly for 2nd order ones, solutions generally are non-smooth and standard approaches in order to define a "weak solution" do not apply: classical, strong almost everywhere, weak, measure-valued and distributional solutions either do not exist or may not even be defined. The main reason for the latter failure is that, the standard idea of using "integration-by-parts" in order to pass derivatives to smooth test functions by duality, is not available for non-divergence structure PDE.
The Navier-Stokes Problem in the 21st Century Pierre Gilles Lemarie-Rieusset 2016-04-06 Up-to-Date Coverage of the Navier–Stokes Equation from an Expert in Harmonic Analysis The complete resolution of the Navier–Stokes equation—one of the Clay Millennium Prize Problems—remains an important open challenge in partial differential equations (PDEs) research despite substantial studies on turbulence and three-dimensional fluids. The Navier–Stokes Problem in the 21st Century provides a self-contained guide to the role of harmonic analysis in the PDEs of fluid mechanics. The book focuses on incompressible deterministic Navier–Stokes equations in the case of a fluid filling the whole space. It explores the meaning of the equations, open problems, and recent progress. It includes classical results on local existence and studies criterion for regularity or uniqueness of solutions. The book also incorporates historical references to the (pre)history of the equations as well as recent references that highlight active mathematical research in the field.
A Minimum Principle for the Smallest Eigenvalue for Second Order Linear Elliptic Equations with Natural Boundary Conditions Charles J. Holland 1977 This paper gives a new characterization of the smallest eigenvalue for second order linear elliptic partial differential equations, not necessarily self-adjoint, with both natural and Dirichlet boundary conditions, and also give a new alternative numerical method for calculating both the smallest eigenvalue and corresponding eigenvector in the case of natural boundary conditions. The smallest eigenvalue, if appropriate sign changes are made, determines the stability of equilibrium solutions to certain second order nonlinear partial differential equations. The corresponding eigenvector enables one to determine the first approximation of the solution of the nonlinear equation to variations of the initial conditions from the equilibrium solution. These nonlinear equations are important in the applications. For these reasons it is important to have these characterizations of the smallest eigenvalue and eigenvector. Our method converts the determination of the eigenvalue and eigenvector to determining the solution of a stationary stochastic control problem. This latter problem is solved and from it a numerical scheme arises naturally. This method appears to have applications in solving other problems.
Stable Solutions of Elliptic Partial Differential Equations Louis Dupaigne 2011-03-15 Stable solutions are ubiquitous in differential equations. They represent meaningful solutions from a physical point of view and appear in many applications, including mathematical physics (combustion, phase transition theory) and geometry (minimal surfaces). Stable Solutions of Elliptic Partial Differential Equations offers a self-contained presentation of the notion of stability in elliptic partial differential equations (PDEs). The central questions of regularity and classification of stable solutions are treated at length. Specialists will find a summary of the most recent developments of the theory, such as nonlocal and higher-order equations. For beginners, the book walks you through the fine versions of the maximum principle, the standard regularity theory for linear elliptic equations, and the fundamental inequalities commonly used in this field. The text also includes two additional topics: the inverse-square potential and some background material on submanifolds of Euclidean space.
Numerical Solution of Partial Differential Equations K. W. Morton 2005-04-11 This is the 2005 second edition of a highly successful and well-respected textbook on the numerical techniques used to solve partial differential equations arising from mathematical models in science, engineering and other fields. The authors maintain an emphasis on finite difference methods for simple but representative examples of parabolic, hyperbolic and elliptic equations from the first edition. However this is augmented by new sections on finite volume methods, modified equation analysis, symplectic integration schemes, convection-diffusion problems, multigrid, and conjugate gradient methods; and several sections, including that on the energy method of analysis, have been extensively rewritten to reflect modern developments. Already an excellent choice for students and teachers in mathematics, engineering and computer science departments, the revised text includes more latest theoretical and industrial developments.
Numerical Methods for Elliptic and Parabolic Partial Differential Equations Peter Knabner 2006-05-26 This text provides an application oriented introduction to the numerical methods for partial differential equations. It covers finite difference, finite element, and finite volume methods, interweaving theory and applications throughout. The book examines modern topics such as adaptive methods, multilevel methods, and methods for convection-dominated problems and includes detailed illustrations and extensive exercises.
Elliptic Partial Differential Equations and Quasiconformal Mappings in the Plane (PMS-48) Kari Astala 2009 This book explores the most recent developments in the theory of planar quasiconformal mappings with a particular focus on the interactions with partial differential equations and nonlinear analysis. It gives a thorough and modern approach to the classical theory and presents important and compelling applications across a spectrum of mathematics: dynamical systems, singular integral operators, inverse problems, the geometry of mappings, and the calculus of variations. It also gives an account of recent advances in harmonic analysis and their applications in the geometric theory of mappings. The book explains that the existence, regularity, and singular set structures for second-order divergence-type equations--the most important class of PDEs in applications--are determined by the mathematics underpinning the geometry, structure, and dimension of fractal sets; moduli spaces of Riemann surfaces; and conformal dynamical systems. These topics are inextricably linked by the theory of quasiconformal mappings. Further, the interplay between them allows the authors to extend classical results to more general settings for wider applicability, providing new and often optimal answers to questions of existence, regularity, and geometric properties of solutions to nonlinear systems in both elliptic and degenerate elliptic settings.
The Cauchy Problem for Solutions of Elliptic Equations Nikolai N. Tarkhanov 1995-05-23 The book is an attempt to bring together various topics in partial differential equations related to the Cauchy problem for solutions of an elliptic equation. Ever since Hadamard, the Cauchy problem for solutions of elliptic equations has been known to be ill-posed. It is conditionally stable, just as is the case for even the simplest problems of analytic continuation of functions given on a subset of the boundary. (Such problems of analytic continuation served as a paradigm for the treatment here.) The study of the Cauchy problem is carried out in three directions: determining the degree of instability, which is connected with sharp theorems on approximation by solutions of an elliptic equation; finding solvability conditions, which is based on the development of Hilbert space methods in the Cauchy problem; and reconstructing solutions via their Cauchy data, which requires efficient ways of approximation. A wide range of topics is touched upon, among them are function spaces on compact sets, boundedness theorems for pseudodifferential operators in nonlocal spaces, nonlinear capacity and removable singularities, fundamental solutions, capacitary criteria for approximation by solutions of elliptic equations, and weak boundary values of the solutions. The theory applies as well to the Cauchy problem for solution of overdetermined elliptic systems.
Contemporary Research in Elliptic PDEs and Related Topics Serena Dipierro 2019-07-12 This volume collects contributions from the speakers at an INdAM Intensive period held at the University of Bari in 2017. The contributions cover several aspects of partial differential equations whose development in recent years has experienced major breakthroughs in terms of both theory and applications. The topics covered include nonlocal equations, elliptic equations and systems, fully nonlinear equations, nonlinear parabolic equations, overdetermined boundary value problems, maximum principles, geometric analysis, control theory, mean field games, and bio-mathematics. The authors are trailblazers in these topics and present their work in a way that is exhaustive and clearly accessible to PhD students and early career researcher. As such, the book offers an excellent introduction to a variety of fundamental topics of contemporary investigation and inspires novel and high-quality research.
On the Asymptotic Stability of Solutions of Nonlinear Partial Differential Equations of Parabolic Type, Part 1 Rangaswamy Narasimhan 1953
Analysis of Finite Difference Schemes Boško S. Jovanović 2013-10-22 This book develops a systematic and rigorous mathematical theory of finite difference methods for linear elliptic, parabolic and hyperbolic partial differential equations with nonsmooth solutions. Finite difference methods are a classical class of techniques for the numerical approximation of partial differential equations. Traditionally, their convergence analysis presupposes the smoothness of the coefficients, source terms, initial and boundary data, and of the associated solution to the differential equation. This then enables the application of elementary analytical tools to explore their stability and accuracy. The assumptions on the smoothness of the data and of the associated analytical solution are however frequently unrealistic. There is a wealth of boundary – and initial – value problems, arising from various applications in physics and engineering, where the data and the corresponding solution exhibit lack of regularity. In such instances classical techniques for the error analysis of finite difference schemes break down. The objective of this book is to develop the mathematical theory of finite difference schemes for linear partial differential equations with nonsmooth solutions. Analysis of Finite Difference Schemes is aimed at researchers and graduate students interested in the mathematical theory of numerical methods for the approximate solution of partial differential equations.
Partial Differential Equations and Boundary-value Problems with Applications Mark A. Pinsky 2011 Building on the basic techniques of separation of variables and Fourier series, the book presents the solution of boundary-value problems for basic partial differential equations: the heat equation, wave equation, and Laplace equation, considered in various standard coordinate systems--rectangular, cylindrical, and spherical. Each of the equations is derived in the three-dimensional context; the solutions are organized according to the geometry of the coordinate system, which makes the mathematics especially transparent. Bessel and Legendre functions are studied and used whenever appropriate throughout the text. The notions of steady-state solution of closely related stationary solutions are developed for the heat equation; applications to the study of heat flow in the earth are presented. The problem of the vibrating string is studied in detail both in the Fourier transform setting and from the viewpoint of the explicit representation (d'Alembert formula). Additional chapters include the numerical analysis of solutions and the method of Green's functions for solutions of partial differential equations. The exposition also includes asymptotic methods (Laplace transform and stationary phase). With more than 200 working examples and 700 exercises (more than 450 with answers), the book is suitable for an undergraduate course in partial differential equations.
Elliptic Differential Equations Wolfgang Hackbusch 1992 Derived from a lecture series for college mathematics students, introduces the methods of dealing with elliptical boundary-value problems--both the theory and the numerical analysis. Includes exercises. Translated and somewhat expanded from the 1987 German version. Annotation copyright by Book News, Inc., Portland, OR
Handbook of Differential Equations:Stationary Partial Differential Equations Michel Chipot 2005-08-19 A collection of self contained, state-of-the-art surveys. The authors have made an effort to achieve readability for mathematicians and scientists from other fields, for this series of handbooks to be a new reference for research, learning and teaching. Partial differential equations represent one of the most rapidly developing topics in mathematics. This is due to their numerous applications in science and engineering on the one hand and to the challenge and beauty of associated mathematical problems on the other. Key features: - Self-contained volume in series covering one of the most rapid developing topics in mathematics. - 7 Chapters, enriched with numerous figures originating from numerical simulations. - Written by well known experts in the field. - Self-contained volume in series covering one of the most rapid developing topics in mathematics. - 7 Chapters, enriched with numerous figures originating from numerical simulations. - Written by well known experts in the field.
Methods for Partial Differential Equations Marcelo R. Ebert 2018-02-23 This book provides an overview of different topics related to the theory of partial differential equations. Selected exercises are included at the end of each chapter to prepare readers for the "research project for beginners" proposed at the end of the book. It is a valuable resource for advanced graduates and undergraduate students who are interested in specializing in this area. The book is organized in five parts: In Part 1 the authors review the basics and the mathematical prerequisites, presenting two of the most fundamental results in the theory of partial differential equations: the Cauchy-Kovalevskaja theorem and Holmgren's uniqueness theorem in its classical and abstract form. It also introduces the method of characteristics in detail and applies this method to the study of Burger's equation. Part 2 focuses on qualitative properties of solutions to basic partial differential equations, explaining the usual properties of solutions to elliptic, parabolic and hyperbolic equations for the archetypes Laplace equation, heat equation and wave equation as well as the different features of each theory. It also discusses the notion of energy of solutions, a highly effective tool for the treatment of non-stationary or evolution models and shows how to define energies for different models. Part 3 demonstrates how phase space analysis and interpolation techniques are used to prove decay estimates for solutions on and away from the conjugate line. It also examines how terms of lower order (mass or dissipation) or additional regularity of the data may influence expected results. Part 4 addresses semilinear models with power type non-linearity of source and absorbing type in order to determine critical exponents: two well-known critical exponents, the Fujita exponent and the Strauss exponent come into play. Depending on concrete models these critical exponents divide the range of admissible powers in classes which make it possible to prove quite different qualitative properties of solutions, for example, the stability of the zero solution or blow-up behavior of local (in time) solutions. The last part features selected research projects and general background material.
Methods on Nonlinear Elliptic Equations Wenxiong Chen 2010
Qualitative Analysis of Nonlinear Elliptic Partial Differential Equations Vicentiu D. Radulescu 2008 This book provides a comprehensive introduction to the mathematical theory of nonlinear problems described by elliptic partial differential equations. These equations can be seen as nonlinear versions of the classical Laplace equation, and they appear as mathematical models in different branches of physics, chemistry, biology, genetics, and engineering and are also relevant in differential geometry and relativistic physics. Much of the modern theory of such equations is based on the calculus of variations and functional analysis. Concentrating on single-valued or multivalued elliptic equations with nonlinearities of various types, the aim of this volume is to obtain sharp existence or nonexistence results, as well as decay rates for general classes of solutions. Many technically relevant questions are presented and analyzed in detail. A systematic picture of the most relevant phenomena is obtained for the equations under study, including bifurcation, stability, asymptotic analysis, and optimal regularity of solutions. The method of presentation should appeal to readers with different backgrounds in functional analysis and nonlinear partial differential equations. All chapters include detailed heuristic arguments providing thorough motivation of the study developed later on in the text, in relationship with concrete processes arising in applied sciences. A systematic description of the most relevant singular phenomena described in this volume includes existence (or nonexistence) of solutions, unicity or multiplicity properties, bifurcation and asymptotic analysis, and optimal regularity. The book includes an extensive bibliography and a rich index, thus allowing for quick orientation among the vast collection of literature on the mathematical theory of nonlinear phenomena described by elliptic partial differential equations.
Wavelet Methods for Elliptic Partial Differential Equations Karsten Urban 2008-11-27 The origins of wavelets go back to the beginning of the last century and wavelet methods are by now a well-known tool in image processing (jpeg2000). These functions have, however, been used successfully in other areas, such as elliptic partial differential equations, which can be used to model many processes in science and engineering. This book, based on the author's course and accessible to those with basic knowledge of analysis and numerical mathematics, gives an introduction to wavelet methods in general and then describes their application for the numerical solution of elliptic partial differential equations. Recently developed adaptive methods are also covered and each scheme is complemented with numerical results, exercises, and corresponding software tools.
Symmetry for Elliptic PDEs Alberto Farina 2010 This volume contains contributions from the INdAM School on Symmetry for Elliptic PDEs, which was held May 25-29, 2009, in Rome, Italy. The school marked ""30 years after a conjecture of De Giorgi, and related problems"" and provided an opportunity for experts to discuss the state of the art and open questions on the subject. Motivated by the classical rigidity properties of the minimal surfaces, De Giorgi proposed the study of the one-dimensional symmetry of the monotone solutions of a semilinear, elliptic partial differential equation. Impressive advances have recently been made in this field, though many problems still remain open. Several generalizations to more complicated operators have attracted the attention of pure and applied mathematicians, both for their important theoretical problems and for their relation, among others, with the gradient theory of phase transitions and the dynamical systems.|This volume contains contributions from the INdAM School on Symmetry for Elliptic PDEs, which was held May 25-29, 2009, in Rome, Italy. The school marked ""30 years after a conjecture of De Giorgi, and related problems"" and provided an opportunity for experts to discuss the state of the art and open questions on the subject. Motivated by the classical rigidity properties of the minimal surfaces, De Giorgi proposed the study of the one-dimensional symmetry of the monotone solutions of a semilinear, elliptic partial differential equation. Impressive advances have recently been made in this field, though many problems still remain open. Several generalizations to more complicated operators have attracted the attention of pure and applied mathematicians, both for their important theoretical problems and for their relation, among others, with the gradient theory of phase transitions and the dynamical systems.
Numerical Solutions for Partial Differential Equations Victor Grigor'e Ganzha 2017-11-22 Partial differential equations (PDEs) play an important role in the natural sciences and technology, because they describe the way systems (natural and other) behave. The inherent suitability of PDEs to characterizing the nature, motion, and evolution of systems, has led to their wide-ranging use in numerical models that are developed in order to analyze systems that are not otherwise easily studied. Numerical Solutions for Partial Differential Equations contains all the details necessary for the reader to understand the principles and applications of advanced numerical methods for solving PDEs. In addition, it shows how the modern computer system algebra Mathematica® can be used for the analytic investigation of such numerical properties as stability, approximation, and dispersion.
Finite Difference Methods for Ordinary and Partial Differential Equations Randall J. LeVeque 2007-01-01 This book introduces finite difference methods for both ordinary differential equations (ODEs) and partial differential equations (PDEs) and discusses the similarities and differences between algorithm design and stability analysis for different types of equations. A unified view of stability theory for ODEs and PDEs is presented, and the interplay between ODE and PDE analysis is stressed. The text emphasizes standard classical methods, but several newer approaches also are introduced and are described in the context of simple motivating examples.
Geometric Properties for Parabolic and Elliptic PDE's Filippo Gazzola 2016-08-08 This book collects recent research papers by respected specialists in the field. It presents advances in the field of geometric properties for parabolic and elliptic partial differential equations, an area that has always attracted great attention. It settles the basic issues (existence, uniqueness, stability and regularity of solutions of initial/boundary value problems) before focusing on the topological and/or geometric aspects. These topics interact with many other areas of research and rely on a wide range of mathematical tools and techniques, both analytic and geometric. The Italian and Japanese mathematical schools have a long history of research on PDEs and have numerous active groups collaborating in the study of the geometric properties of their solutions.
Elliptic Partial Differential Equations Vitaly Volpert 2014-05-10 If we had to formulate in one sentence what this book is about, it might be "How partial differential equations can help to understand heat explosion, tumor growth or evolution of biological species". These and many other applications are described by reaction-diffusion equations. The theory of reaction-diffusion equations appeared in the first half of the last century. In the present time, it is widely used in population dynamics, chemical physics, biomedical modelling. The purpose of this book is to present the mathematical theory of reaction-diffusion equations in the context of their numerous applications. We will go from the general mathematical theory to specific equations and then to their applications. Existence, stability and bifurcations of solutions will be studied for bounded domains and in the case of travelling waves. The classical theory of reaction-diffusion equations and new topics such as nonlocal equations and multi-scale models in biology will be considered.
Stable Solutions of Elliptic Partial Differential Equations Louis Dupaigne 2011-03-15 Stable solutions are ubiquitous in differential equations. They represent meaningful solutions from a physical point of view and appear in many applications, including mathematical physics (combustion, phase transition theory) and geometry (minimal surfaces). Stable Solutions of Elliptic Partial Differential Equations offers a self-contained presentation of the notion of stability in elliptic partial differential equations (PDEs). The central questions of regularity and classification of stable solutions are treated at length. Specialists will find a summary of the most recent developments of the theory, such as nonlocal and higher-order equations. For beginners, the book walks you through the fine versions of the maximum principle, the standard regularity theory for linear elliptic equations, and the fundamental functional inequalities commonly used in this field. The text also includes two additional topics: the inverse-square potential and some background material on submanifolds of Euclidean space.
Numerical Solution of Elliptic Differential Equations by Reduction to the Interface Boris N. Khoromskij 2012-12-06 During the last decade essential progress has been achieved in the analysis and implementation of multilevel/rnultigrid and domain decomposition methods to explore a variety of real world applications. An important trend in mod ern numerical simulations is the quick improvement of computer technology that leads to the well known paradigm (see, e. g. , [78,179]): high-performance computers make it indispensable to use numerical methods of almost linear complexity in the problem size N, to maintain an adequate scaling between the computing time and improved computer facilities as N increases. In the h-version of the finite element method (FEM), the multigrid iteration real izes an O(N) solver for elliptic differential equations in a domain n c IRd d with N = O(h-) , where h is the mesh parameter. In the boundary ele ment method (BEM) , the traditional panel clustering, fast multi-pole and wavelet based methods as well as the modern hierarchical matrix techniques are known to provide the data-sparse approximations to the arising fully populated stiffness matrices with almost linear cost O(Nr log?Nr), where 1 d Nr = O(h -) is the number of degrees of freedom associated with the boundary. The aim of this book is to introduce a wider audience to the use of a new class of efficient numerical methods of almost linear complexity for solving elliptic partial differential equations (PDEs) based on their reduction to the interface.
Fast Direct Solvers for Elliptic PDEs Per-Gunnar Martinsson 2019-12-16 Fast solvers for elliptic PDEs form a pillar of scientific computing. They enable detailed and accurate simulations of electromagnetic fields, fluid flows, biochemical processes, and much more. This textbook provides an introduction to fast solvers from the point of view of integral equation formulations, which lead to unparalleled accuracy and speed in many applications. The focus is on fast algorithms for handling dense matrices that arise in the discretization of integral operators, such as the fast multipole method and fast direct solvers. While the emphasis is on techniques for dense matrices, the text also describes how similar techniques give rise to linear complexity algorithms for computing the inverse or the LU factorization of a sparse matrix resulting from the direct discretization of an elliptic PDE. This is the first textbook to detail the active field of fast direct solvers, introducing readers to modern linear algebraic techniques for accelerating computations, such as randomized algorithms, interpolative decompositions, and data-sparse hierarchical matrix representations. Written with an emphasis on mathematical intuition rather than theoretical details, it is richly illustrated and provides pseudocode for all key techniques. Fast Direct Solvers for Elliptic PDEs is appropriate for graduate students in applied mathematics and scientific computing, engineers and scientists looking for an accessible introduction to integral equation methods and fast solvers, and researchers in computational mathematics who want to quickly catch up on recent advances in randomized algorithms and techniques for working with data-sparse matrices.
The obstacle problem Luis Angel Caffarelli 1999-10-01 The material presented here corresponds to Fermi lectures that I was invited to deliver at the Scuola Normale di Pisa in the spring of 1998. The obstacle problem consists in studying the properties of minimizers of the Dirichlet integral in a domain D of Rn, among all those configurations u with prescribed boundary values and costrained to remain in D above a prescribed obstacle F. In the Hilbert space H1(D) of all those functions with square integrable gradient, we consider the closed convex set K of functions u with fixed boundary value and which are greater than F in D. There is a unique point in K minimizing the Dirichlet integral. That is called the solution to the obstacle problem.
Nonlinear Partial Differential Equations for Future Applications Shigeaki Koike 2021-04-16 This volume features selected, original, and peer-reviewed papers on topics from a series of workshops on Nonlinear Partial Differential Equations for Future Applications that were held in 2017 at Tohoku University in Japan. The contributions address an abstract maximal regularity with applications to parabolic equations, stability, and bifurcation for viscous compressible Navier–Stokes equations, new estimates for a compressible Gross–Pitaevskii–Navier–Stokes system, singular limits for the Keller–Segel system in critical spaces, the dynamic programming principle for stochastic optimal control, two kinds of regularity machineries for elliptic obstacle problems, and new insight on topology of nodal sets of high-energy eigenfunctions of the Laplacian. This book aims to exhibit various theories and methods that appear in the study of nonlinear partial differential equations.
PETSc for Partial Differential Equations: Numerical Solutions in C and Python Ed Bueler 2020-10-22 The Portable, Extensible Toolkit for Scientific Computation (PETSc) is an open-source library of advanced data structures and methods for solving linear and nonlinear equations and for managing discretizations. This book uses these modern numerical tools to demonstrate how to solve nonlinear partial differential equations (PDEs) in parallel. It starts from key mathematical concepts, such as Krylov space methods, preconditioning, multigrid, and Newton's method. In PETSc these components are composed at run time into fast solvers. Discretizations are introduced from the beginning, with an emphasis on finite difference and finite element methodologies. The example C programs of the first 12 chapters, listed on the inside front cover, solve (mostly) elliptic and parabolic PDE problems.

Discretization leads to large, sparse, and generally nonlinear systems of algebraic equations. For such problems, mathematical solver concepts are explained and illustrated through the examples, with sufficient context to speed further development. PETSc for Partial Differential Equations addresses both discretizations and fast solvers for PDEs, emphasizing practice more than theory. Well-structured examples lead to run-time choices that result in high solver performance and parallel scalability. The last two chapters build on the reader's understanding of fast solver concepts when applying the Firedrake Python finite element solver library. This textbook, the first to cover PETSc programming for nonlinear PDEs, provides an on-ramp for graduate students and researchers to a major area of high-performance computing for science and engineering. It is suitable as a supplement for courses in scientific computing or numerical methods for differential equations.

The Concept of Stability in Numerical Mathematics Wolfgang Hackbusch 2014-02-06 In this book, the author compares the meaning of stability in different subfields of numerical mathematics. Concept of Stability in numerical mathematics opens by examining the stability of finite algorithms. A more precise definition of stability holds for quadrature and interpolation methods, which the following chapters focus on. The discussion then progresses to the numerical treatment of ordinary differential equations (ODEs). While one-step methods for ODEs are always stable, this is not the case for hyperbolic or parabolic differential equations, which are investigated next. The final chapters discuss stability for discretisations of elliptic differential equations and integral equations. In comparison among the subfields we discuss the practical importance of stability and the possible conflict between higher consistency order and stability.

Numerical Analysis of Partial Differential Equations S. H, Lui 2012-01-10 A balanced guide to the essential techniques for solving elliptic partial differential equations Numerical Analysis of Partial Differential Equations provides a comprehensive, self-contained treatment of the quantitative methods used to solve elliptic partial differential equations (PDEs), with a focus on the efficiency as well as the error of the presented methods. The author utilizes coverage of theoretical PDEs, along with the nu merical solution of linear systems and various examples and exercises, to supply readers with an introduction to the essential concepts in the numerical analysis of PDEs. The book presents the three main discretization methods of elliptic PDEs: finite difference, finite elements, and spectral methods. Each topic has its own devoted chapters and is discussed alongside additional key topics, including: The mathematical theory of elliptic PDEs Numerical linear algebra Time-dependent PDEs Multigrid and domain decomposition PDEs posed on infinite domains The book concludes with a discussion of the methods for nonlinear problems, such as Newton's method, and addresses the importance of hands-on work to facilitate learning. Each chapter concludes with a set of exercises, including theoretical and programming problems, that allows readers to test their understanding of the presented theories and techniques. In addition, the book discusses important nonlinear problems in many fields of science and engineering, providing information as to how they can serve as computing projects across various disciplines. Requiring only a preliminary understanding of analysis, Numerical Analysis of Partial Differential Equations is suitable for courses on numerical PDEs at the upper-undergraduate and graduate levels. The book is also appropriate for students majoring in the mathematical sciences and engineering.

Partial Differential Equations Walter A. Strauss 2007-12-21 Partial Differential Equations presents a balanced and comprehensive introduction to the concepts and techniques required to solve problems containing unknown functions of multiple variables. While focusing on the three most classical partial differential equations (PDEs)—the wave, heat, and Laplace equations—this detailed text also presents a broad practical perspective that merges mathematical concepts with real-world application in diverse areas including molecular structure, photon and electron interactions, radiation of electromagnetic waves, vibrations of a solid, and many more. Rigorous pedagogical tools aid in student comprehension; advanced topics are introduced frequently, with minimal technical jargon, and a wealth of exercises reinforce vital skills and invite additional self-study. Topics are presented in a logical progression, with major concepts such as wave propagation, heat and diffusion, electrostatics, and quantum mechanics placed in contexts familiar to students of various fields in science and engineering. By understanding the properties and applications of PDEs, students will be equipped to better analyze and interpret central processes of the natural world.

Partial Differential Equations of Applied Mathematics Erich Zauderer 2011-10-24 This new edition features the latest tools for modeling, characterizing, and solving partial differential equations The Third Edition of this classic text offers a comprehensive guide to modeling, characterizing, and solving partial differential equations (PDEs). The author provides all the theory and tools necessary to solve problems via exact, approximate, and numerical methods. The Third Edition retains all the hallmarks of its previous editions, including an emphasis on practical applications, clear writing style and logical organization, and extensive use of real-world examples. Among the new and revised material, the book features: * A new section at the end of each original chapter, exhibiting the use of specially constructed Maple procedures that solve PDEs via many of the methods presented in the chapters. The results can be evaluated numerically or displayed graphically. * Two new chapters that present finite difference and finite element methods for the solution of PDEs. Newly constructed Maple procedures are provided and used to carry out each of these methods. All the numerical results can be displayed graphically. * A related FTP site that includes all the Maple code used in the text. * New exercises in each chapter, and answers to many of the exercises are provided via the FTP site. A supplementary Instructor's Solutions Manual is available. The book begins with a demonstration of how the three basic types of equations-parabolic, hyperbolic, and elliptic-can be derived from random walk models. It then covers an exceptionally broad range of topics, including questions of stability, analysis of singularities, transform methods, Green's functions, and perturbation and asymptotic treatments. Approximation methods for simplifying complicated problems and solutions are described, and linear and nonlinear problems not easily solved by standard methods are examined in depth. Examples from the fields of engineering and physical sciences are used liberally throughout the text to help illustrate how theory and techniques are applied to actual problems. With its extensive use of examples and exercises, this text is recommended for advanced undergraduates and graduate students in engineering, science, and applied mathematics, as well as professionals in any of these fields. It is possible to use the text, as in the past, without use of the new Maple material.

Handbook of Differential Equations: Stationary Partial Differential Equations Michel Chipot 2011-08-11 This handbook is the sixth and last volume in the series devoted to stationary partial differential equations. The topics covered by this volume include in particular domain perturbations for boundary value problems, singular solutions of semilinear elliptic problems, positive solutions to elliptic equations on unbounded domains, symmetry of solutions, stationary compressible Navier-Stokes equation, Lotka-Volterra systems with cross-diffusion, and fixed point theory for elliptic boundary value problems. * Collection of self-contained, state-of-the-art surveys * Written by well-known experts in the field * Informs and updates on all the latest developments